2023年度數學Mock卷英文版答案參考及常犯錯誤報告

Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles . All fractional answers must be simplified.

Paper 1

Solution	Marks	Remarks
1. $\dfrac{m^2n^6}{(m^{-5}n^4)^{-3}}$		
$=\dfrac{m^2n^6}{m^{15}n^{-12}}$	1M	for $(a^h)^k=a^{hk}$ or $(ab)^l=a^lb^l$
$=\dfrac{n^{6+12}}{m^{15-2}}$	1M	for $\dfrac{c^p}{c^q}=c^{p-q}$ or $d^{-r}=\dfrac{1}{d^r}$
$=\dfrac{n^{18}}{m^{13}}$	1A	
	----------(3)	
2. $u=\dfrac{vw+1}{2-w}$		
$u(2-w)=vw+1$	1M	
$2u-uw=vw+1$		
$-uw-vw=1-2u$	1M	for putting w on one side
$-w(u+v)=1-2u$		
$w=\dfrac{2u-1}{u+v}$	1A	or equivalent
	----------(3)	
3. (a) $18x^2-50y^2$		
$=2(9x^2-25y^2)$		
$=2(3x-5y)(3x+5y)$	1A	or equivalent
(b) $9x-15y-18x^2+50y^2$		
$=9x-15y-2(3x-5y)(3x+5y)$	1M	for using the result of (a)
$=3(3x-5y)-2(3x-5y)(3x+5y)$		
$=(3x-5y)(3-6x-10y)$	1A	or equivalent
	----------(3)	
4. (a) 16.8	1A	
(b) 16.734	1A	
(c) 17	1A	
	----------(3)	

Solution	Marks	Remarks
5. Let \$$x$ and \$$y$ be the price of a pen case and the total price of a pencil sharpener and a hole puncher respectively.		
Then, we have $x+5y=363$ and $3x+2y=322$.	1A+1A	
$2x-5(3x)=2(363)-5(322)$	1M	for getting a linear equation in x or y only
$-13x=-884$		
$x=68$		
$y=59$		
The required price		
$=4(68)+59$		
$=\$331$	1A	
Let \$$x$ be the price of a pen case.		
$3x+2\left(\frac{363-x}{5}\right)=322$	1M+1A+1A	$\begin{cases} 1\text{A for } y=\frac{363-x}{5} \\ +\ 1\text{M for } 3x+2y \end{cases}$
$\frac{13x}{5}+\frac{726}{5}=322$		
$x=68$		
The required price		
$=4(68)+\frac{363-68}{5}$		
$=\$331$	1A	
	----------(4)	
6. (a) $\frac{5x+22}{3}\geq 2(x+3)$		
$5x+22\geq 6x+18$		
$5x-6x\geq 18-22$	1M	for putting x on one side
$-x\geq -4$		
$x\leq 4$	1A	
(b) $9-4x<0$		
$x>\frac{9}{4}$	1A	$x>2.25$
By (a), we have $\frac{9}{4}<x\leq 4$.		
Thus, the required number is 2 .	1A	
	----------(4)	

Solution	Marks	Remarks
7. Note that $\angle ABE = \angle ADE$, $\angle DBE = \angle DAE$ and $\angle ADE = \angle DAE$.	1M	for either one
So, we have $\angle ABE = \angle ADE = \angle DAE = \angle DBE = 32°$.		
Let F be the point of intersection of AC and BD .		
$\angle BAC$		
$= 180° - \angle AFB - \angle ABD$		
$= 180° - 90° - (32° + 32°)$		
$= 26°$	1A	
Since BD is the perpendicular bisector of AC , BD passes through the centre of the circle.		
Hence, BD is a diameter of the circle.		
Note that $\angle BAD = 90°$.	1A	
$\angle CAE$		
$= \angle DAE + \angle BAD - \angle BAC$		
$= 32° + 90° - 26°$		
$= 96°$		
$\angle CDE$		
$= 180° - \angle CAE$	1M	
$= 180° - 96°$		
$= 84°$	1A	
Note that $\angle ABE = \angle ADE$, $\angle DBE = \angle DAE$ and $\angle ADE = \angle DAE$.	1M	for either one
So, we have $\angle ABE = \angle ADE = \angle DAE = \angle DBE = 32°$.		
Let F be the point of intersection of AC and BD .		
$\angle BAC$		
$= 180° - \angle AFB - \angle ABD$		
$= 180° - 90° - (32° + 32°)$		
$= 26°$	1A	
Since BD is the perpendicular bisector of AC , BD passes through the centre of the circle.		
Hence, BD is a diameter of the circle.		
Note that $\angle BED = 90°$.	1A	
$\angle BDE$		
$= 180° - \angle BED - \angle DBE$	1M	
$= 180° - 90° - 32°$		
$= 58°$		
Note that $\angle BAC = \angle BDC = 26°$.		
$\angle CDE$		
$= \angle BDE + \angle BDC$		
$= 58° + 26°$		
$= 84°$	1A	
	----------(5)	

Solution	Marks	Remarks
8. (a) The median		
= 53 thousand dollars	1A	
The range		
= 86 − 32		
= 54 thousand dollars	1A	
The inter-quartile range		
= 65 − 41		
= 24 thousand dollars	1A	
(b) Note that the median of the monthly rents of the shops of the shopping mall after the redecoration is 66 thousand dollars which is greater than the upper quartile of the monthly rents of the shops of the shopping mall before the redecoration.	1M	
Thus, the claim is agreed.	1A	f.t.
	----------(5)	
9. (a) Let a cm and h cm be the radius of the sphere and the height of the circular cone respectively.		
$9\left(\frac{4}{3}\pi a^3\right)=\frac{1}{3}\pi(3a)^2h$	1M	
$h=4a$		
Thus, the required ratio is 4 : 1 .	1A	
(b) Let a cm be the radius of a sphere.		
$4\pi a^2=576\pi$	1M	
$a=12$		
So, the base radius and the height of the circular cone are 36 cm and 48 cm respectively.		
The total surface area of the circular cone		
$=\pi(36)^2+\pi(36)\sqrt{36^2+48^2}$	1M	
$=3\ 456\pi\ \text{cm}^2$	1A	
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $f(x) = p(x+2)^2 + q$, where p and q are non-zero constants.	1M	
Since $f(0) = -45$ and $f(3) = 39$, we have $4p + q = -45$ and $25p + q = 39$.	1M	for either substitution
Solving, we have $p = 4$ and $q = -61$.		
Thus, we have $f(x) = 4(x+2)^2 - 61$.	1A	
	----------(3)	
(b) $f(x) = 13x$		
$4(x+2)^2 - 61 = 13x$		
$4x^2 + 3x - 45 = 0$	1M	
$(x-3)(4x+15) = 0$		
$x = 3$ or $x = -\frac{15}{4}$	1A	$x = 3$ or $x = -3.75$
	----------(2)	
11. (a) $\frac{25+20+a+27+29+34+36+38(3)+42+44+(40+b)(2)+47+51+55}{16} = 39$	1M	
$a + 2b = 20$		
Note that $5 \le a \le 7$ and $4 \le b \le 7$.		
Thus, we have $a = 6$ and $b = 7$.	1A	for both correct
	----------(2)	
(b) (i) Since the original distribution has two modes and their frequencies are both 3 , the frequency of the mode of the new distribution is greater than 3 .	1M	
Thus, we have $c = 38$ or $c = 47$.	1A	for both correct
(ii) By (b)(i), there are two cases.		
Case 1: $c = 38$ The standard deviation ≈ 8.412219214	1M	either one
Case 2: $c = 47$ The standard deviation ≈ 8.774260899		
Thus, the greatest possible standard deviation of the distribution of the ages of the teachers attending the seminar is 8.77 .	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
12. (a) Let $ax+b$ be the required quotient, where a and b are constants.	1M	
Then, we have $f(x)=(x^2-x-2)(ax+b)+(15x+35)$.		
Note that the coefficients of x^2 and x in the expansion of		
$(x^2-x-2)(ax+b)+(15x+35)$ are $b-a$ and $15-2a-b$ respectively.		
So, we have $b-a=8$ and $15-2a-b=-11$.	1M	
Solving, we have $a=6$ and $b=14$.		
Thus, the required quotient is $6x+14$.	1A	
Let $f(x)=(x^2-x-2)g(x)+(15x+35)$, where $g(x)$ is a polynomial.	1M	
Hence, we have $f(2)=((2)^2-(2)-2)g(2)+(15(2)+35)=65$ and		
$f(-1)=((-1)^2-(-1)-2)g(-1)+(15(-1)+35)=20$.	1M	
Note that $f(2)=h(2)^3+8(2)^2-11(2)+k=8h+k+10$ and		
$f(-1)=h(-1)^3+8(-1)^2-11(-1)+k=k-h+19$.		
So, we have $8h+k+10=65$ and $k-h+19=20$.		
Solving, we have $h=6$ and $k=7$.		
$f(x)$		
$=6x^3+8x^2-11x+7$		
$=(x^2-x-2)(6x+14)+(15x+35)$		
Thus, the required quotient is $6x+14$.	1A	
	----------(3)	
(b) $f(x)=0$		
$(x^2-x-2)(6x+14)+(15x+35)=0$		
$(x^2-x-2)(2(3x+7))+5(3x+7)=0$		
$(3x+7)(2x^2-2x+1)=0$	1M	for $(3x+7)(px^2+qx+r)$
$x=-\frac{7}{3}$ or $2x^2-2x+1=0$		
$(-2)^2-4(2)(1)$	1M	
$=-4$		
<0		
So, the quadratic equation $2x^2-2x+1=0$ does not have real roots.	1M	
Note that $-\frac{7}{3}$ is a rational root of the equation $f(x)=0$.		
Thus, the equation $f(x)=0$ has 1 rational root.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
13. **Marking Schemes for (a)(i) and (a)(ii) :**		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(a) (i) $\angle AFC = \angle BFD$ (common $\angle$)		
$\angle CAF = \angle DBF$ (given)		
$\angle ACF = \angle BDF$ ($\angle$ sum of Δ)		
$\Delta ACF \sim \Delta BDF$ (AAA)		(AA) (equiangular)
(ii) By (a)(i), we have $\frac{AF}{BF} = \frac{CF}{DF}$. (corr. sides, ~ Δs)		
So, we have $\frac{AF}{CF} = \frac{BF}{DF}$.		
$\angle AFB = \angle CFD$ (common $\angle$)		
$\Delta ABF \sim \Delta CDF$ (ratio of 2 sides, inc. $\angle$)		
$\angle ABC = \angle CDE$ (corr. $\angle$s, ~ Δs)		
$\angle ACB = \angle ECD$ (given)		
$\angle BAC = \angle DEC$ ($\angle$ sum of Δ)		
$\Delta ABC \sim \Delta EDC$ (AAA)	----------(4)	(AA) (equiangular)
(b) (i) $\frac{CE}{AC} = \frac{CD}{BC}$	1M	
$\frac{CE}{52} = \frac{25}{65}$		
$CE = 20$ cm		
$AE^2 + CE^2$		
$= 48^2 + 20^2$		
$= 2\ 704$		
$= 52^2$		
$= AC^2$		
Therefore, $\angle AEC$ is a right angle.		
Thus, ΔCDE is a right-angled triangle.	1A	f.t.
(ii) By (a)(ii), we have $\angle BAC = \angle DEC = 90°$.		
AB		
$= \sqrt{BC^2 - AC^2}$	1M	
$= \sqrt{65^2 - 52^2}$		
$= 39$ cm		
The area of ΔABC		
$= \frac{1}{2}(AB)(AC)$		
$= \frac{1}{2}(39)(52)$		
$= 1\ 014\ \text{cm}^2$	1A ----------(4)	

Solution	Marks	Remarks
14. (a) Let (h, k) be the coordinates of G.		
Then, we have $2h + 5k - 161 = 0$.		
$AG = BG$		
$\sqrt{(h-9)^2 + (k-26)^2} = \sqrt{(h-33)^2 + (k-44)^2}$	1M	
$h^2 - 18h + 81 + k^2 - 52k + 676 = h^2 - 66h + 1\ 089 + k^2 - 88k + 1\ 936$		
$4h + 3k - 189 = 0$		
Solving, we have $h = 33$ and $k = 19$.		
The equation of C is		
$(x-33)^2 + (y-19)^2 = (9-33)^2 + (26-19)^2$	1M	
$(x-33)^2 + (y-19)^2 = 625$	1A	$x^2 + y^2 - 66x - 38y + 825 = 0$
	----------(3)	
(b) (i) Γ is the perpendicular bisector of the line segment AB.	1M	
(ii) (1) Let d be the y-coordinate of D.		
The slope of AB $= \dfrac{44-26}{33-9} = \dfrac{3}{4}$		
Note that the slope of DG is $\dfrac{19-d}{33-0}$.		
Hence, we have $\left(\dfrac{19-d}{33-0}\right)\left(\dfrac{3}{4}\right) = -1$.	1M	
Solving, we have $d = 63$.		
The distance between D and G $= \sqrt{(0-33)^2 + (63-19)^2}$		
$= 55$	1A	

Solution	Marks	Remarks
(2) AB		
$=\sqrt{(33-9)^2+(44-26)^2}$		
$=30$		
Let M be the mid-point of AB .		
Note that $AM=15$.		
Also note that the radius of C is 25 .		
GM		
$=\sqrt{AG^2-AM^2}$	1M	
$=\sqrt{25^2-15^2}$		
$=20$		
The area of ΔABE		
$=\frac{1}{2}(AB)(EM)$	1M	either one
$=\frac{1}{2}(AB)(EG-GM)$		
$=\frac{1}{2}(30)(25-20)$		
$=75$		
The area of ΔADE		
$=\frac{1}{2}(DE)(AM)$		
$=\frac{1}{2}(DG-EG)(AM)$		
$=\frac{1}{2}(55-25)(15)$		
$=225$		
The required ratio		
$=75:225$		
$=1:3$	1A	
	----------(6)	

Solution	Marks	Remarks
15. (a) The required probability		
$=\dfrac{C_1^1C_5^6}{C_6^{11}}$	1M	for numerator
$=\dfrac{1}{77}$	1A	r.t. 0.0130
The required probability		
$=6\left(\dfrac{1}{11}\right)\left(\dfrac{6}{10}\right)\left(\dfrac{5}{9}\right)\left(\dfrac{4}{8}\right)\left(\dfrac{3}{7}\right)\left(\dfrac{2}{6}\right)$	1M	for $6p_1p_2p_3p_4p_5p_6$
$=\dfrac{1}{77}$	1A	r.t. 0.0130
	----------(2)	
(b) The required probability		
$=1-\dfrac{1}{77}-\dfrac{C_1^1C_1^4C_4^6}{C_6^{11}}-\dfrac{C_1^1C_4^4C_1^6}{C_6^{11}}-\dfrac{C_6^6}{C_6^{11}}$	1M	for $1-(a)-p_7-p_8-p_9$
$=\dfrac{389}{462}$	1A	r.t. 0.842
The required probability		
$=1-\dfrac{1}{77}-30\left(\dfrac{1}{11}\right)\left(\dfrac{4}{10}\right)\left(\dfrac{6}{9}\right)\left(\dfrac{5}{8}\right)\left(\dfrac{4}{7}\right)\left(\dfrac{3}{6}\right)-30\left(\dfrac{1}{11}\right)\left(\dfrac{4}{10}\right)\left(\dfrac{3}{9}\right)\left(\dfrac{2}{8}\right)\left(\dfrac{1}{7}\right)\left(\dfrac{6}{6}\right)$ $-\left(\dfrac{6}{11}\right)\left(\dfrac{5}{10}\right)\left(\dfrac{4}{9}\right)\left(\dfrac{3}{8}\right)\left(\dfrac{2}{7}\right)\left(\dfrac{1}{6}\right)$	1M	for $1-(a)-p_{10}-p_{11}-p_{12}$
$=\dfrac{389}{462}$	1A	r.t. 0.842
The required probability		
$=\dfrac{C_6^{10}}{C_6^{11}}-\dfrac{C_6^6}{C_6^{11}}+\dfrac{C_1^1C_2^4C_3^6}{C_6^{11}}+\dfrac{C_1^1C_3^4C_2^6}{C_6^{11}}$	1M	for $p_{13}-p_{14}+p_{15}+p_{16}$
$=\dfrac{389}{462}$	1A	r.t. 0.842
The required probability		
$=\left(\dfrac{10}{11}\right)\left(\dfrac{9}{10}\right)\left(\dfrac{8}{9}\right)\left(\dfrac{7}{8}\right)\left(\dfrac{6}{7}\right)\left(\dfrac{5}{6}\right)-\left(\dfrac{6}{11}\right)\left(\dfrac{5}{10}\right)\left(\dfrac{4}{9}\right)\left(\dfrac{3}{8}\right)\left(\dfrac{2}{7}\right)\left(\dfrac{1}{6}\right)$ $+60\left(\dfrac{1}{11}\right)\left(\dfrac{4}{10}\right)\left(\dfrac{3}{9}\right)\left(\dfrac{6}{8}\right)\left(\dfrac{5}{7}\right)\left(\dfrac{4}{6}\right)+60\left(\dfrac{1}{11}\right)\left(\dfrac{4}{10}\right)\left(\dfrac{3}{9}\right)\left(\dfrac{2}{8}\right)\left(\dfrac{6}{7}\right)\left(\dfrac{5}{6}\right)$	1M	for $p_{17}-p_{18}+p_{19}+p_{20}$
$=\dfrac{389}{462}$	1A	r.t. 0.842
	----------(2)	

Solution	Marks	Remarks
16. (a) Let d be the common difference of the arithmetic sequence.		
So, we have $\frac{6}{2}(2(\text{A}(1))+(6-1)d)=246$ and $\frac{15}{2}(2(\text{A}(1))+(15-1)d)=345$.	1M	for either one
Hence, we have $2(\text{A}(1))+5d=82$ and $\text{A}(1)+7d=23$.		
Solving, we have $d=-4$.		
Thus, we have $\text{A}(1)=51$.	1A	
	----------(2)	
(b) $\text{G}(1)\text{G}(2)\text{G}(3)\cdots\text{G}(k)>2\ 023$		
$\log_{0.9}\text{G}(1)+\log_{0.9}\text{G}(2)+\log_{0.9}\text{G}(3)+\cdots+\log_{0.9}\text{G}(k)<\log_{0.9}2\ 023$	1M	
Note that $\log_{0.9}\text{G}(n)=\text{A}(n)$ for all positive integers n .		
$\text{A}(1)+\text{A}(2)+\text{A}(3)+\cdots+\text{A}(k)<\log_{0.9}2\ 023$		
$\frac{k}{2}(2(51)+(k-1)(-4))<\log_{0.9}2\ 023$		
$2k^2-53k+\log_{0.9}2\ 023>0$		
$k<-1.299490982$ or $k>27.79949098$	1M	
Thus, the least value of k is 28 .	1A	
	----------(3)	

Solution	Marks	Remarks
17. (a) Putting $y = mx$ into $x^2 + y^2 + 12x - 10y + 25 = 0$, we have		
$x^2 + (mx)^2 + 12x - 10(mx) + 25 = 0$	1M	
$(m^2 + 1)x^2 + (-10m + 12)x + 25 = 0$		
Since L and C intersect at two distinct points, we have		
$(-10m + 12)^2 - 4(m^2 + 1)(25) > 0$	1M	
$-240m + 44 > 0$		
$m < \frac{11}{60}$	1A	
	----------(3)	
(b) Since Q and R lie on C and the circumcentre of ΔPQR is the centre of C , P lies on C .		
Since P , Q and R are not collinear, P does not lie on L for each value of m in the range found in (a).		
Denote the origin by O .		
Note that L is the straight line passing through the O with slope m .		
There are two cases.		
Case 1: The straight line OP is a not vertical line.		
Note that the straight line OP intersects C at only one point.		
By (a), the slope of OP is $\frac{11}{60}$.		
$\left(\left(\frac{11}{60}\right)^2 + 1\right)x^2 + \left(-10\left(\frac{11}{60}\right) + 12\right)x + 25 = 0$	1M	
$3\,721x^2 + 36\,600x + 90\,000 = 0$		
$(61x + 300)^2 = 0$		
$x = -\frac{300}{61}$		
Putting $x = -\frac{300}{61}$ into $y = \frac{11}{60}x$, we have $y = -\frac{55}{61}$.		
Therefore, the coordinates of P are $\left(-\frac{300}{61}, -\frac{55}{61}\right)$.		
Case 2: The straight line OP is a vertical line.		
Putting $x = 0$ into $x^2 + y^2 + 12x - 10y + 25 = 0$, we have		
$y^2 - 10y + 25 = 0$	1M	
$(y - 5)^2 = 0$		
$y = 5$		
Therefore, the coordinates of P are $(0, 5)$.		
Thus, the coordinates of P are $\left(-\frac{300}{61}, -\frac{55}{61}\right)$ or $(0, 5)$.	1A	for both correct
	----------(3)	

Solution	Marks	Remarks
18. (a) Since A lies on the graph of $y = \mathrm{f}(x)$, we have $-(2k)^2 + 6k(2k) + c = 0$.	1M	
Thus, we have $c = -8k^2$.	1A	
	----------(2)	
(b) $\mathrm{f}(x)$		
$= -x^2 + 6kx - 8k^2$		
$= -(x^2 - 6kx) - 8k^2$		
$= -(x^2 - 6kx + 9k^2 - 9k^2) - 8k^2$	1M	
$= -(x - 3k)^2 + k^2$		
Thus, the coordinates of Q are $(3k, k^2)$.	1A	
	----------(2)	
(c) (i) $\left(3k, \dfrac{-k^2}{9}\right)$	1A	
(ii) Note that the straight line $x = 3k$ is perpendicular to AB and passes through Q .		
So, the orthocentre of ΔABQ lies on the straight line $x = 3k$.		
Hence, we have $18 = 3k$.		
Solving, we have $k = 6$.	1A	
Note that the coordinates of A , Q and R are $(12, 0)$, $(18, 36)$ and $(18, -4)$ respectively.		
The product of the slope of AQ and the slope of AR		
$=\left(\dfrac{36-0}{18-12}\right)\left(\dfrac{(-4)-0}{18-12}\right)$		
$= -4$		
$\neq -1$		
Hence, we have $\angle QAR \neq 90°$.	1M	
Note that B is the reflection image of A with respect to the straight line QR .		
So, we have $\angle QBR = \angle QAR$.		
Hence, we have $\angle QAR + \angle QBR \neq 180°$.		
Thus, A , B , Q are R are not concyclic.	1A	f.t.

Solution	Marks	Remarks
Note that the straight line $x = 3k$ is perpendicular to AB and passes through Q . So, the orthocentre of ΔABQ lies on the straight line $x = 3k$. Hence, we have $18 = 3k$. Solving, we have $k = 6$.	1A	
Note that the coordinates of A , Q and R are (12, 0) , (18, 36) and (18, –4) respectively. $AQ^2 + AR^2$ $= ((18-12)^2 + (36-0)^2) + ((18-12)^2 + ((-4)-0)^2)$ $= 1\ 384$ QR^2 $= (36-(-4))^2$ $= 1\ 600$ So, we have $AQ^2 + AR^2 \neq QR^2$. Hence, we have $\angle QAR \neq 90°$.	1M	
Note that B is the reflection image of A with respect to the straight line QR . So, we have $\angle QBR = \angle QAR$. Hence, we have $\angle QAR + \angle QBR \neq 180°$. Thus, A , B , Q are R are not concyclic.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
19. (a) By sine formula, we have $\dfrac{IL}{\sin\angle IKL}=\dfrac{IK}{\sin\angle ILK}$	1M	either one
By sine formula, we have $\dfrac{JL}{\sin\angle JKL}=\dfrac{JK}{\sin\angle JLK}$		
Hence, we have $\dfrac{IL\sin\angle JKL}{JL\sin\angle IKL}=\dfrac{IK\sin\angle JLK}{JK\sin\angle ILK}$.		
Since $\angle ILK+\angle JLK=180^\circ$, we have $\sin\angle ILK=\sin\angle JLK$.		
Thus, we have $\dfrac{\sin\angle IKL}{\sin\angle JKL}=\dfrac{(IL)(JK)}{(JL)(IK)}$.	1	
The area of ΔIKL $=\dfrac{1}{2}(IK)(KL)\sin\angle IKL$	1M	either one
The area of ΔJKL $=\dfrac{1}{2}(JK)(KL)\sin\angle JKL$		
Note that $\dfrac{\text{The area of }\Delta IKL}{\text{The area of }\Delta JKL}=\dfrac{IL}{JL}$.		
Hence, we have $\dfrac{IK\sin\angle IKL}{JK\sin\angle JKL}=\dfrac{IL}{JL}$.		
Thus, we have $\dfrac{\sin\angle IKL}{\sin\angle JKL}=\dfrac{(IL)(JK)}{(JL)(IK)}$.	1	
	----------(2)	
(b) (i) Let E be the point of intersection of AC and BD .		
$BE=AB\cos\angle ABE=10\cos 45^\circ=5\sqrt{2}$ cm		
VE $=\sqrt{VB^2-BE^2}$ $=\sqrt{17^2-(5\sqrt{2})^2}$ $=\sqrt{239}$ cm		
Note that $AE=BE=5\sqrt{2}$ cm and $AC=2BE=10\sqrt{2}$ cm .		
By cosine formula, we have $\cos\angle VAC=\dfrac{VA^2+AE^2-VE^2}{2(VA)(AE)}$	1M	either one
$\cos\angle VAC=\dfrac{18^2+(5\sqrt{2})^2-(\sqrt{239})^2}{2(18)(5\sqrt{2})}$		
$\cos\angle VAC=\dfrac{3\sqrt{2}}{8}$		
By cosine formula, we have $VC^2=VA^2+AC^2-2(VA)(AC)\cos\angle VAC$		
$VC^2=18^2+(10\sqrt{2})^2-2(18)(10\sqrt{2})\left(\dfrac{3\sqrt{2}}{8}\right)$		
$VC=\sqrt{254}$ cm	1A	r.t. 15.9 cm

Solution	Marks	Remarks
(ii) (1) Note that AQ is the perpendicular bisector of PR . Let S be the point of intersection of AQ and PR . Note that $\Delta PVS \sim \Delta BVE$. So, we have $\dfrac{VS}{VE}=\dfrac{VP}{VB}$.	1M	
Hence, we have $VS : ES = 3 : 2$.		
By (a), we have $\dfrac{\sin\angle VAQ}{\sin\angle CAQ}=\dfrac{(VQ)(AC)}{(CQ)(VA)}$ and $\dfrac{\sin\angle VAQ}{\sin\angle CAQ}=\dfrac{(VS)(AE)}{(ES)(VA)}$.		
So, we have $\dfrac{(VQ)(AC)}{(CQ)(VA)}=\dfrac{(VS)(AE)}{(ES)(VA)}$.	1M	
Hence, we have $\dfrac{VQ}{CQ}=\left(\dfrac{AE}{AC}\right)\left(\dfrac{VS}{ES}\right)=\left(\dfrac{1}{2}\right)\left(\dfrac{3}{2}\right)=\dfrac{3}{4}$.		
Thus, the required ratio is 3 : 4 .	1A	
(2) Since $\cos\angle VAC=\dfrac{3\sqrt{2}}{8}$, we have $\sin\angle VAC=\dfrac{\sqrt{46}}{8}$. The area of ΔAVC		
$=\dfrac{1}{2}(VA)(AC)\sin\angle VAC$	1M	
$=\dfrac{1}{2}(18)(10\sqrt{2})\left(\dfrac{\sqrt{46}}{8}\right)$ $=\dfrac{45\sqrt{23}}{2}$ cm^2		
Note that $\dfrac{\text{The area of }\Delta AVQ}{\text{The area of }\Delta AVC}=\dfrac{VQ}{VC}$. Hence, the area of ΔAVQ is $\dfrac{135\sqrt{23}}{14}$ cm^2 .		
Note that $\Delta PVS \sim \Delta BVE$. So, we have $\dfrac{PS}{BE}=\dfrac{VP}{VB}$.	1M	
Hence, we have $PS=3\sqrt{2}$ cm .		
The volume of the pyramid $VAPQR$ = 2(the volume of the pyramid $VAPQ$) $=2\left(\dfrac{1}{3}(\text{the area of }\Delta AVQ)(PS)\right)$ $=2\left(\dfrac{1}{3}\left(\dfrac{135\sqrt{23}}{14}\right)(3\sqrt{2})\right)$		
$=\dfrac{135\sqrt{46}}{7}$ cm^3	1A	r.t. 131 cm^3

Solution	Marks	Remarks
(3) The angle between VA and the plane $APQR$ and the angle between the plane $APQR$ and the plane $ABCD$ are $\angle VAQ$ and $\angle CAQ$ respectively.	1M	for either one
$\dfrac{\sin\angle VAQ}{\sin\angle CAQ}$ $=\dfrac{(VQ)(AC)}{(CQ)(VA)}$ $=\left(\dfrac{3}{4}\right)\left(\dfrac{10\sqrt{2}}{18}\right)$ $=\dfrac{5\sqrt{2}}{12}$ ≤ 1 Hence, we have $\sin\angle VAQ \leq \sin\angle CAQ$. Since $\angle VAQ$ and $\angle CAQ$ are acute angles, we have $\angle VAQ \leq \angle CAQ$. Thus, the claim is disagreed.	1A	f.t.
The angle between VA and the plane $APQR$ and the angle between the plane $APQR$ and the plane $ABCD$ are $\angle VAQ$ and $\angle CAQ$ respectively.	1M	for either one
$\dfrac{\sin\angle VAQ}{\sin\angle CAQ}$ $=\dfrac{(VS)(AE)}{(ES)(VA)}$ $=\left(\dfrac{3}{2}\right)\left(\dfrac{5\sqrt{2}}{18}\right)$ $=\dfrac{5\sqrt{2}}{12}$ ≤ 1 Hence, we have $\sin\angle VAQ \leq \sin\angle CAQ$. Since $\angle VAQ$ and $\angle CAQ$ are acute angles, we have $\angle VAQ \leq \angle CAQ$. Thus, the claim is disagreed.	1A	f.t.
	----------(10)	

Paper 2

Question No.	Key	Question No.	Key
1.	A	26.	A
2.	D	27.	A
3.	D	28.	A
4.	B	29.	A
5.	D	30.	C
6.	A	31.	C
7.	B	32.	D
8.	C	33.	B
9.	C	34.	A
10.	D	35.	B
11.	C	36.	C
12.	C	37.	A
13.	A	38.	C
14.	B	39.	D
15.	B	40.	D
16.	D	41.	C
17.	C	42.	D
18.	B	43.	A
19.	C	44.	B
20.	C	45.	B
21.	D		
22.	D		
23.	B		
24.	B		
25.	A		

1. A

$$\frac{(3^{5n})(9^{3n})}{81^{2n}}=\frac{(3^{5n})[(3^2)^{3n}]}{(3^4)^{2n}}=\frac{(3^{5n})(3^{6n})}{3^{8n}}=3^{3n}=(3^3)^n=27^n$$

2. D

$$\begin{aligned}\frac{1}{p}+\frac{3}{2+q}&=4\\ \frac{3}{2+q}&=4-\frac{1}{p}\\ &=\frac{4p-1}{p}\\ 2+q&=\frac{3p}{4p-1}\\ q&=\frac{3p}{4p-1}-2\\ &=\frac{3p-2(4p-1)}{4p-1}\\ &=\frac{2-5p}{4p-1}\end{aligned}$$

3. D

$$\begin{aligned}(x+y-1)(x-y+1)&=[x+(y-1)][x-(y-1)]\\ &=x^2-(y-1)^2\\ &=x^2-(y^2-2y+1)\\ &=x^2-y^2+2y-1\end{aligned}$$

4. B

$$\begin{aligned}\frac{4}{3x+2}-\frac{1}{3x-2}&=\frac{4(3x-2)-(3x+2)}{(3x+2)(3x-2)}\\ &=\frac{12x-8-3x-2}{9x^2-4}\\ &=\frac{9x-10}{9x^2-4}\end{aligned}$$

5. D

$$\begin{array}{rcr}\dfrac{4x-3}{7}\le x-3 & \text{or} & 5x-11>19\\ 4x-3\le 7x-21 & \text{or} & 5x>30\\ -3x\le -18 & \text{or} & x>6\\ x\ge 6 & \text{or} & x>6\end{array}$$

Thus, the required solution is $x \ge 6$.

6. A

By remainder theorem, we have

$$\begin{aligned}f\left(-\frac{2}{3}\right)&=0\\ 3k\left(-\frac{2}{3}\right)^4+13\left(-\frac{2}{3}\right)^3+8\left(-\frac{2}{3}\right)+4k&=0\\ \frac{124}{27}k-\frac{248}{27}&=0\\ k&=2\end{aligned}$$

By remainder theorem, we have

$$\begin{aligned}\text{The required remainder} &= f(-1)\\ &= 6(-1)^4 + 13(-1)^3 + 8(-1) + 8\\ &= -7\end{aligned}$$

7. B

Method 1

Putting $x = 2$ into the identity, we have

$$\begin{aligned}(2)^2 - 3m(2) + n &= [(2) - 8][(2) + m] - 3n\\ 4 - 6m + n &= -12 - 6m - 3n\\ n &= -4\end{aligned}$$

Method 2

$x^2 - 3mx + n \equiv (x - 8)(x + m) - 3n$
$x^2 - 3mx + n \equiv x^2 + (m - 8)x + (-8m - 3n)$

Comparing the coefficient of the x term and the constant term, we have

$$\begin{cases}-3m = m - 8\\ n = -8m - 3n\end{cases}$$

$$\begin{cases}m = 2\\ 2m + n = 0\end{cases}$$

Solving, we have $m = 2$ and $n = -4$.

8. C

$$\begin{aligned}f(f(k)f(-k)) &= f((k^2 + 2k - k^2)[(-k)^2 + 2(-k) - k^2])\\ &= f((2k)(-2k))\\ &= f(-4k^2)\\ &= (-4k^2)^2 + 2(-4k^2) - k^2\\ &= 16k^4 - 8k^2 - k^2\\ &= 16k^4 - 9k^2\end{aligned}$$

9. C

$$\begin{aligned}y &= (m - x)(x + n) - mn\\ &= (mx - x^2 + mn - nx) - mn\\ &= -x^2 + (m - n)x\end{aligned}$$

I ✓ The coefficient of $x^2 = -1 < 0$
Thus, the graph opens downwards.

II ✗ The y-intercept of the graph $= 0$
Thus, the y-intercept of the graph is not negative.

III ✓ Put $(x, y) = (m - n, 0)$ into $y = -x^2 + (m - n)x$.
L.H.S. $= 0$ and R.H.S. $= -(m - n)^2 + (m - n)(m - n) = 0$
Thus, the graph passes through the point $(m - n, 0)$.

10. D

Let $\$C$ be the cost of the bookshelf.

Then the marked price of the bookshelf is $\$C \times (1 + x\%)$.
Hence, the selling price of the bookshelf is $\$C \times (1 + x\%) \times (1 - 25\%)$.

$$\begin{aligned}C \times (1 + x\%) \times (1 - 25\%) &= C \times (1 + 20\%)\\ x &= 60\end{aligned}$$

11. C

The amount $= 18000 \times (1 + 3\% \div 2)^{5\times 2}$
≈ 20889.73485
$\approx \$20890$ (corr. to the nearest dollar)

12. C

Note that the region on the map and the actual region are similar.

$$\frac{\text{The area of the lake on the map}}{\text{The actual area of the lake}} = \left(\frac{1}{25000}\right)^2 = \frac{1}{6.25\times 10^8}$$

Thus, we have

The area of the lake on the map $= 2.4 \div (6.25 \times 10^8)$ km^2
$= [2.4 \times (1000 \times 100)^2] \div (6.25 \times 10^8)$ cm^2
$= 38.4$ cm^2

13. A

Let $z = \dfrac{ax^3}{\sqrt{y}}$, where a is a non-zero constant.

Then, we have $a = \dfrac{\sqrt{y}z}{x^3}$.

Let p be the percentage change in z when x is increased by 20% and y is increased by 125%.
Then we have

$$\frac{\sqrt{(1+125\%)y}[(1+p)z]}{[(1+20\%)x]^3} = \frac{\sqrt{y}z}{x^3}$$
$$1 + p = 1.152$$
$$p = 0.152$$

Thus, z is increased by 15.2%.

14. B

Let $T(m)$ be the number of dots in the mth pattern.

Note that (1): $T(1) = 2$
(2): $T(n + 1) = T(n) + 3$ for any positive integer n

$T(12) = T(11) + 3$ (by (2) with $n = 11$)
$= T(10) + 3 + 3$ (by (2) with $n = 10$)
$\vdots$
$= T(1) + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$ (by (2) with $n = 1$)
$= 2 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$ (by (1))
$= 35$

15. B

Note that 4.5 cm $\le AB <$ 5.5 cm, 8.5 cm $\le BC <$ 9.5 cm, 2.5 cm $\le EF <$ 3.5 cm and 2.5 cm $\le AF <$ 3.5 cm.

Let G be the point of intersection of AB and DE produced.

The area of the hexagon = The area of the rectangle $AGEF$ + The area of the rectangle $BCDG$
$= EF \cdot AF + BG \cdot BC$
$= EF \cdot AF + (AB - EF) \cdot BC$

Hence, the area of the hexagon increases when AB, BC or AF increases.

Let H be the point of intersection of AF produced and CD produced.

The area of the hexagon = The area of the rectangle $ABCH$ − The area of the rectangle $DEFH$
$= AB \cdot BC - EF \cdot FH$
$= AB \cdot BC - EF \cdot (BC - AF)$

Hence, the area of the hexagon decreases when EF increases.

Therefore, we have

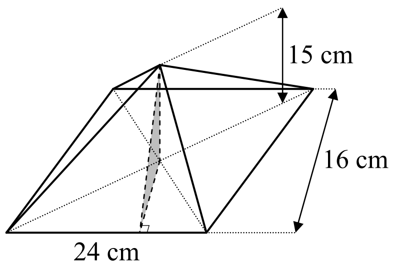
$$(3.5)(2.5) + (4.5 - 3.5)(8.5) < x < (2.5)(3.5) + (5.5 - 2.5)(9.5)$$
$$17.25 < x < 37.25$$

16. D

Let b cm be the perpendicular distance between the opposite sides of the base.

Consider the volume of the pyramid.

$$\frac{1}{3}[(24)b](15) = 1920$$
$$b = 16$$



The height of the lateral faces of the pyramid $= \sqrt{\left(\frac{16}{2}\right)^2 + 15^2} = 17$ cm

The total surface area of the pyramid $= (24)(16) + 4 \cdot \frac{1}{2}(24)(17) = 1\,200$ cm^2

17. C

Since ΔBDE and ΔADE have common height, we have

$$\frac{\text{The area of } \Delta BDE}{\text{The area of } \Delta ADE} = \frac{EB}{AE} = \frac{3}{4}$$

Since ΔBDF and ΔBDE have common height, we have

$$\frac{\text{The area of } \Delta BDF}{\text{The area of } \Delta BDE} = \frac{BF}{EB} = 1$$

In ΔADF and ΔBGF,

$\angle ADF = \angle BGF$ (corr. $\angle$s, $AD \,//\, BC$)
$\angle DAF = \angle GBF$ (corr. $\angle$s, $AD \,//\, BC$)
$\angle AFD = \angle BFG$ (common $\angle$)

Hence, we have $\Delta ADF \sim \Delta BGF$. (AAA)

$AF : BF = (AE + EB + BF) : BF = (4 + 3 + 3) : 3 = 10 : 3$

$DF : GF = AF : BF = 10 : 3$ (corr. sides, $\sim \Delta$s)

Since ΔBDG and ΔBDF have common height, we have

$$\frac{\text{The area of } \Delta BDG}{\text{The area of } \Delta BDF} = \frac{DG}{DF} = \frac{7}{10}$$

Therefore, we have

$$\begin{aligned}\text{The area of the quadrilateral } BEDG &= \text{The area of } \Delta BDE + \text{The area of } \Delta BDG \\ &= \text{The area of } \Delta ADE \times \frac{3}{4} \times \left(1 + 1 \times \frac{7}{10}\right) \\ &= 40 \times \frac{51}{40} \\ &= 51 \text{ cm}^2\end{aligned}$$

18. B

$AE = BE$ (given)

$\angle BAE = \angle ABE$ (base $\angle$s, isos. Δ)

$\angle BAE + \angle ABE = 180° - \angle AEB = 180° - 24° = 156°$ ($\angle$ sum of Δ)

Therefore, we have $\angle BAE = 156° \div 2 = 78°$.

Note that $BCDE$ is a parallelogram.

$$\begin{aligned} CD &= BE && \text{(prop. of //gram)} \\ &= AC && \text{(given)} \\ \angle DAC &= \angle ADC && \text{(base } \angle\text{s, isos. } \Delta) \\ &= \angle AEB && \text{(corr. } \angle\text{s, } CD \mathbin{//} BE) \\ &= 24° \end{aligned}$$

$\angle BAC = \angle BAE - \angle DAC = 78° - 24° = 54°$

19. C

$\angle ABC = \angle BAC$ (given)

$BC = AC = 36$ cm (sides opp. equal $\angle$s)

In ΔBCD and ΔDCE,

$$\begin{aligned} \angle CBD &= \angle CDE && \text{(given)} \\ \angle BCD &= \angle DCE && \text{(common } \angle) \\ \angle BDC &= \angle DEC && (\angle \text{ sum of } \Delta) \end{aligned}$$

Hence, we have $\Delta BCD \sim \Delta DCE$. (AAA)

$$\begin{aligned} \frac{CD}{CE} &= \frac{BC}{DC} && \text{(corr. sides, } \sim \Delta\text{s)} \\ \frac{30}{CE} &= \frac{36}{30} \\ CE &= 25 \text{ cm} \end{aligned}$$

$BE = BC - CE = 36 - 25 = 11$ cm

In ΔACD,
$$\begin{aligned} \angle ACD + \angle CAD &= \angle CDB && \text{(ext. } \angle \text{ of } \Delta) \\ &= \angle CDE + \angle BDE \\ \angle ACD &= \angle BDE \end{aligned}$$

In ΔACD and ΔBDE,

$$\begin{aligned} \angle CAD &= \angle DBE && \text{(given)} \\ \angle ACD &= \angle BDE && \text{(proved)} \\ \angle ADC &= \angle BED && (\angle \text{ sum of } \Delta) \end{aligned}$$

Hence, we have $\Delta ACD \sim \Delta BDE$. (AAA)

$$\begin{aligned} \frac{AD}{BE} &= \frac{CD}{DE} && \text{(corr. sides, } \sim \Delta\text{s)} \\ \frac{AD}{11} &= \frac{30}{15} \\ AD &= 22 \text{ cm} \end{aligned}$$

20. C

$BC = AD = 26$ cm (prop. of //gram)

$AB^2 + AC^2 = 10^2 + 24^2 = 676$

$BC^2 = 26^2 = 676$

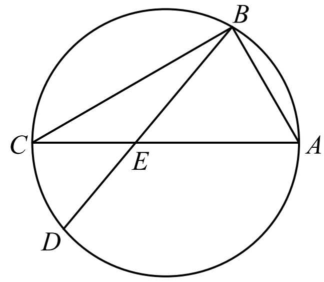
Hence, we have $AB^2 + AC^2 = BC^2$.

Therefore, we have $\angle BAC = 90°$. (converse of Pyth. theorem)

The area of the parallelogram $ABCD = AB \cdot AC = (10)(24) = 240 \text{ cm}^2$

21. D

Since AC is a diameter of the circle $ABCD$, we have



$$\begin{aligned}\overset{\frown}{ABC} &= \overset{\frown}{ADC}\\ \overset{\frown}{AB}+\overset{\frown}{BC} &= \overset{\frown}{AD}+\overset{\frown}{CD}\\ \frac{3}{2}\overset{\frown}{CD}+\frac{6}{2}\overset{\frown}{CD} &= \overset{\frown}{AD}+\overset{\frown}{CD}\\ \overset{\frown}{AD} &= \frac{7}{2}\overset{\frown}{CD}\end{aligned}$$

Therefore, we have $\overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{CD} : \overset{\frown}{AD} = 3 : 6 : 2 : 7$.

$\angle ACB : \angle BAC : \angle CBD : \angle ABD = 3 : 6 : 2 : 7$ (arcs prop. to $\angle$s at circumference)

Let $\angle ACB = 3x$, $\angle BAC = 6x$, $\angle CBD = 2x$ and $\angle ABD = 7x$.

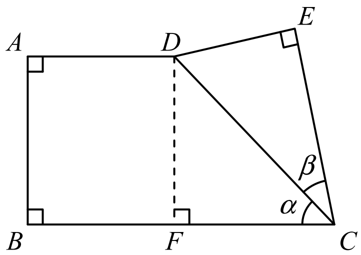
$$\begin{aligned}\angle BAC + \angle ABC + \angle ACB &= 180^\circ \quad (\angle \text{ sum of } \Delta)\\ 6x + (2x + 7x) + 3x &= 180^\circ\\ x &= 10^\circ\end{aligned}$$

$\angle BEC = \angle BAC + \angle ABD = 60^\circ + 70^\circ = 130^\circ$ (ext. $\angle$ of Δ)

22. D

Let F be the foot of perpendicular from D to BC.

Note that $ABFD$ is a rectangle.

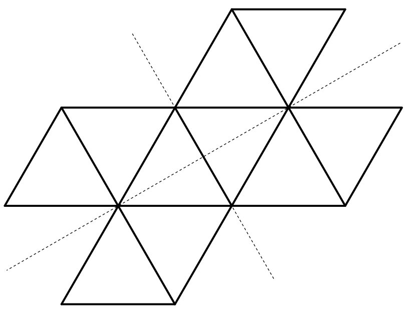


$$\begin{aligned}\frac{DF}{CD} &= \sin\angle DCF\\ \frac{AB}{CD} &= \sin\alpha \text{ .... (1)}\\ \frac{CE}{CD} &= \cos\angle DCE\\ &= \cos\beta \text{ .... (2)}\end{aligned}$$

(1) ÷ (2): $\dfrac{AB}{CE} = \dfrac{\sin\alpha}{\cos\beta}$

23. B

The number of axes of reflectional symmetry of the figure is 2. (See the figure below.)



24. B

Note that L is perpendicular to the perpendicular bisector of AB.

$$\begin{aligned}\text{The slope of } L \times \text{The slope of the perpendicular bisector of } AB &= -1\\ \left(-\frac{3}{-1}\right)\times\left(-\frac{k}{-12}\right) &= -1\\ k &= -4\end{aligned}$$

The x-intercept of $L = -\dfrac{h}{3}$ and The y-intercept of $L = -\dfrac{h}{-1} = h$

So, the coordinates of A and B are $\left(-\dfrac{h}{3}, 0\right)$ and $(0, h)$ respectively.

The coordinates of the mid-point of $AB = \left(\dfrac{1}{2}\left(-\dfrac{h}{3}+0\right), \dfrac{1}{2}(0+h)\right) = \left(-\dfrac{h}{6}, \dfrac{h}{2}\right)$

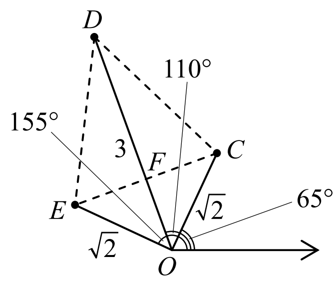
Note that the mid-point of AB lies on the perpendicular bisector of AB.

$$(-4)\left(-\frac{h}{6}\right) - 12\left(\frac{h}{2}\right) + 3(-4)^2 = 0$$
$$h = 9$$

The x-intercept of $L = -\dfrac{9}{3} = -3$

25. A

Denote the pole by O.



$\angle COD = 110° - 65° = 45°$

$\angle DOE = 155° - 110° = 45°$

Note that $OC = OE$ and OD bisects $\angle COE$.

Hence, OD is the perpendicular bisector of CE. (prop. of isos. Δ)

Let F be the point of intersection of CE and OD.

$CF = OC \sin\angle COF = \sqrt{2}\sin 45° = 1$

$OF = OC \cos\angle COF = \sqrt{2}\cos 45° = 1$

$CE = 2CF = 2$

$DF = OD - OF = 3 - 1 = 2$

The area of $\Delta CDE = \dfrac{1}{2} CE \cdot DF = \dfrac{1}{2}(2)(2) = 2$

26. A

$AB = \sqrt{[(-2)-3]^2 + (3-5)^2} = \sqrt{29}$

$CD = \sqrt{[(-4)-1]^2 + [(-3)-(-1)]^2} = \sqrt{29}$

Since the area of ΔPAB is equal to the area of ΔPCD and $AB = CD$, the perpendicular distance from P to AB is equal to the perpendicular distance from P to CD.

The slope of $AB = \dfrac{3-5}{(-2)-3} = \dfrac{2}{5}$

The slope of $BC = \dfrac{(-1)-3}{1-(-2)} = -\dfrac{4}{3}$

The slope of $CD = \dfrac{(-3)-(-1)}{(-4)-1} = \dfrac{2}{5}$

So, the straight lines AB and CD are two parallel lines.

The locus of a moving point equidistant from two parallel lines is a straight line.

27. A

I ✓ Let G be the centre of C.

$$\frac{x^2}{4}+\frac{y^2}{4}+8x-12y+39=0$$
$$x^2+y^2+32x-48y+156=0$$

The coordinates of $G=\left(-\frac{32}{2},-\frac{-48}{2}\right)=(-16,\ 24)$

The radius of $C=\sqrt{\left(\frac{32}{2}\right)^2+\left(\frac{-48}{2}\right)^2-156}=26$

$OG=\sqrt{[0-(-16)]^2+(0-24)^2}=\sqrt{832}$

Note that OG is greater than the radius of C.

Thus, O lies outside C.

II ✗ The perpendicular distance from G to the x-axis $=24-0=24$

Note that the perpendicular distance from G to the x-axis is smaller than the radius of C.

Hence, C cuts the x-axis at two distinct points.

Thus, C does not lie in the second quadrant.

III ✗ The perpendicular distance from G to the y-axis $=0-(-16)=16$

Note that the perpendicular distance from G to the y-axis is smaller than the radius of C.

Hence, C cuts the y-axis at two distinct points.

Consider the case when P is a point of intersection between C and the x-axis and Q is a point of intersection between C and the y-axis.

$\angle POQ$ is equal to a right angle.

Thus, $\angle POQ$ is not smaller than a right angle for this case.

28. A

There are 36 possible outcomes, namely {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {1, 7}, {1, 8}, {1, 9}, {2, 3}, {2, 4}, {2, 5}, {2, 6}, {2, 7}, {2, 8}, {2, 9}, {3, 4}, {3, 5}, {3, 6}, {3, 7}, {3, 8}, {3, 9}, {4, 5}, {4, 6}, {4, 7}, {4, 8}, {4, 9}, {5, 6}, {5, 7}, {5, 8}, {5, 9}, {6, 7}, {6, 8}, {6, 9}, {7, 8}, {7, 9} and {8, 9}.

There are 14 favourable outcomes, namely {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {1, 7}, {1, 8}, {1, 9}, {2, 4}, {2, 6}, {2, 8}, {3, 6}, {3, 9} and {4, 8}.

The required probability $=\frac{14}{36}=\frac{7}{18}$

29. A

Since there are 18 data, the lower quartile is the 5th ordered datum and the upper quartile is the 14th ordered datum.

The lower quartile of the distribution = 3
The upper quartile of the distribution = 5

The inter-quartile range of the distribution $=5-3=2$

30. C

The sum of the 15 integers is $120 + 2k$.

Consider the values of a, b and c according to the value of k.

k	a (mean)	b (median)	c (mode)	$a \geq b$	$a \geq c$	$b \geq c$
7	$\frac{134}{15}$	9	7	✗	✓	✓
8	$\frac{136}{15}$	9	8	✓	✓	✓
9	$\frac{46}{5}$	9	9	✓	✓	✓
			Combining	✗	✓	✓

31. C

$$\begin{aligned}3 \times 64^5 - 7 \times 128^3 + 11 \times 256 &= 3 \times (2^6)^5 - 7 \times (2^7)^3 + 11 \times 2^8 \\ &= 3 \times 2^{30} - 7 \times 2^{21} + 11 \times 2^8 \\ &= 3 \times 2^{4 \times 7 + 2} - 7 \times 2^{4 \times 5 + 1} + 11 \times 2^{4 \times 2} \\ &= 3 \times 2^2 \times (2^4)^7 - 7 \times 2^1 \times (2^4)^5 + 11 \times (2^4)^2 \\ &= 12 \times 16^7 - 14 \times 16^5 + 11 \times 16^2 \\ &= 11 \times 16^7 + (16^2 - 14) \times 16^5 + 11 \times 16^2 \\ &= 11 \times 16^7 + (15 \times 16 + 16 - 14) \times 16^5 + 11 \times 16^2 \\ &= 11 \times 16^7 + (15 \times 16 + 2) \times 16^5 + 11 \times 16^2 \\ &= 11 \times 16^7 + 15 \times 16^6 + 2 \times 16^5 + 11 \times 16^2 \\ &= \mathrm{BF200B00}_{16}\end{aligned}$$

32. D

I ✓ When $x = 0$, since the graph of $y = kb^x$ lies below the graph of $y = a^x$, we have $kb^0 < a^0$.
Thus, we have $k < 1$.

II ✓ Since the graph of $y = a^x$ is downward sloping, a^x is a decreasing function of x.
Hence, we have $0 < a < 1$.
Since the graph of $y = kb^x$ is upward sloping, kb^x is an increasing function of x.
Since $k > 0$, b^x is an increasing function of x.
Hence, we have $b > 1$.
Thus, we have $a < b$.

III ✓ When $x \geq k$, since the graph of $y = a^x$ lies below the graph of $y = kb^x$, we have $a^x < kb^x$.
Since $1 > k$, we have $a^1 < kb^1$.
Thus, we have $a < kb$.

33. B

By the relations between roots and coefficients, we have $\alpha + \beta = -\frac{m}{1} = -m$.

$$\begin{aligned}\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= (\alpha + \beta)\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) - \alpha - \beta \\ &= (\alpha + \beta)\left[\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) - 1\right] \\ &= (-m)\left[\left(\frac{m}{2} - 2\right) - 1\right] \\ &= \frac{6m - m^2}{2}\end{aligned}$$

34. A

I ✓ $a_6 a_7 a_8 a_9 a_{10} = -64$

$a_8^{\ 5} = -64$

$a_8 = -2^{\frac{6}{5}}$

Therefore, a_8 is an irrational number.

Thus, some of the terms of the sequence are irrational numbers.

II ✓ Let r be the common ratio of the geometric sequence.

Note that $a_2 = \dfrac{a_8}{r^6}$.

Since $a_8 < 0$ and $r^6 > 0$, we have $a_2 < 0$.

III ✗ Note that $a_1 + a_2 = a_1 + a_1 r = a_1(1 + r)$.

Since $a_1 > 0$ and $a_1 + a_2 > 0$, we have $1 + r > 0$.

Thus, we have $r > -1$.

35. B

I ✗ $u = \dfrac{1}{ki+2} = \dfrac{1}{ki+2} \cdot \dfrac{-ki+2}{-ki+2} = \dfrac{-ki+2}{k^2+4}$ and $v = \dfrac{1}{ki-2} = \dfrac{1}{ki-2} \cdot \dfrac{-ki-2}{-ki-2} = \dfrac{-ki-2}{k^2+4}$

So, the real part of u and the real part of v are $\dfrac{2}{k^2+4}$ and $\dfrac{-2}{k^2+4}$ respectively.

Thus, the real part of u is not equal to the real part of v.

II ✓ $\dfrac{1}{u} = ki + 2$ and $\dfrac{1}{v} = ki - 2$

So, the imaginary parts of $\dfrac{1}{u}$ and $\dfrac{1}{v}$ are both k.

Thus, the imaginary part of $\dfrac{1}{u}$ is equal to the imaginary part of $\dfrac{1}{v}$.

III ✗ $u + v = \dfrac{-ki+2}{k^2+4} + \dfrac{-ki-2}{k^2+4} = \dfrac{-2ki}{k^2+4}$

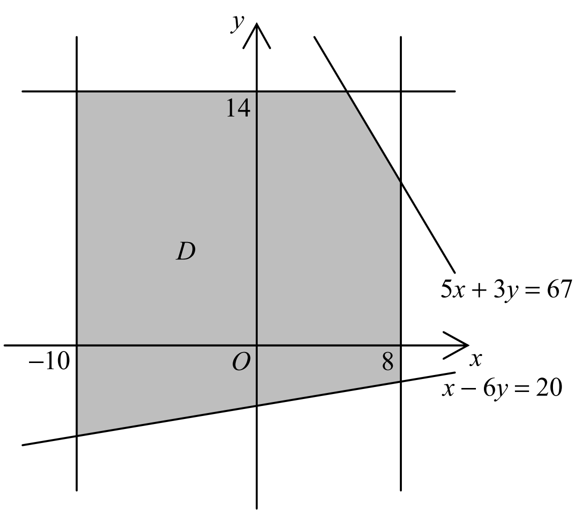
Since k is an irrational number, we have $k \neq 0$.

Hence, $u + v$ is not a real number.

Thus, $u + v$ is not an irrational number.

36. C

The figure below shows D.



Solving $x = -10$ and $y = 14$, we have $(x, y) = (-10, 14)$.
Solving $y = 14$ and $5x + 3y = 67$, we have $(x, y) = (5, 14)$.
Solving $5x + 3y = 67$ and $x = 8$, we have $(x, y) = (8, 9)$.
Solving $x = 8$ and $x - 6y = 20$, we have $(x, y) = (8, -2)$.
Solving $x - 6y = 20$ and $x = -10$, we have $(x, y) = (-10, -5)$.

Note that the greatest value is attained at a corner point.

At $(x, y) = (-10, 14)$, $4x - 5y = 4(-10) - 5(14) = -110$
At $(x, y) = (5, 14)$, $4x - 5y = 4(5) - 5(14) = -50$
At $(x, y) = (8, 9)$, $4x - 5y = 4(8) - 5(9) = -13$
At $(x, y) = (8, -2)$, $4x - 5y = 4(8) - 5(-2) = 42$
At $(x, y) = (-10, -5)$, $4x - 5y = 4(-10) - 5(-5) = -15$

Thus, the greatest value of $4x - 5y$ is 42.

37. A

$$\begin{aligned} 13\sin x + 5\cos^2 x &= 11 \\ 13\sin x + 5(1 - \sin^2 x) &= 11 \\ 5\sin^2 x - 13\sin x + 6 &= 0 \\ (5\sin x - 3)(\sin x - 2) &= 0 \end{aligned}$$

$\sin x = \dfrac{3}{5}$ or $\sin x = 2$ (rejected)

$x \approx 36.86989765°$ or $143.1301024°$

Thus, the equation has 2 roots.

38. C

I. ✓ Let $AB = (x + 14)$ cm.

Then we have $AC = (x - 14)$ cm.

By Heron's formula, we have

$s = \dfrac{AB + AC + BC}{2} = \dfrac{(x+14)+(x-14)+36}{2} = (x + 18)$ cm

$$\begin{aligned} \sqrt{s(s - AB)(s - AC)(s - BC)} &= 176 \\ s(s - AB)(s - AC)(s - BC) &= 30976 \\ (x + 18)[(x + 18) - (x + 14)][(x + 18) - (x - 14)][(x + 18) - 36] &= 30976 \\ (x + 18)(4)(32)(x - 18) &= 30976 \\ (x + 18)(x - 18) &= 242 \\ x^2 - 324 &= 242 \\ x^2 &= 566 \\ x &= \sqrt{566} \quad \text{or} \quad -\sqrt{566} \quad \text{(rejected)} \end{aligned}$$

Therefore, we have $AB = (\sqrt{566} + 14)$ cm and $AC = (\sqrt{566} - 14)$ cm.

Note that $AB \approx 37.79075451$ cm < 40 cm.

Thus, the length of the longest side of ΔABC is smaller than 40 cm.

II ✗ $BC - AC = 36 - (\sqrt{566} - 14) \approx 26.20924549$ cm

$AB - BC = (\sqrt{566} + 14) - 36 \approx 1.790754507$ cm

Hence, we have $BC - AC \neq AB - BC$.

Thus, the lengths of the sides of ΔABC cannot form an arithmetic sequence.

III. ✓ By cosine formula, we have

$$\cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$$

$$= \frac{(\sqrt{566} - 14)^2 + 36^2 - (\sqrt{566} + 14)^2}{2(\sqrt{566} - 14)(36)}$$

$$\angle ACB \approx 92.95025902^\circ$$
$$> 90^\circ$$

Thus, ΔABC is an obtuse-angled triangle.

39. D

Let R be a point on the straight line PQ such that $AEBR$ is a straight line.

$\angle BCR = \angle BAC = 19^\circ$ ($\angle$ in alt. segment)

$\angle ACB = 90^\circ$ ($\angle$ in semi-circle)

$$\angle ARC = 180^\circ - \angle CAR - \angle ACR \quad (\angle \text{ sum of } \Delta)$$
$$= 180^\circ - 19^\circ - (90^\circ + 19^\circ)$$
$$= 52^\circ$$

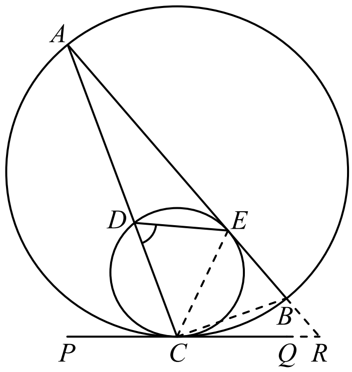
$CR = ER$ (tangent properties)

$\angle CER = \angle ECR$ (base $\angle$s, isos. Δ)

$\angle CER + \angle ECR = 180^\circ - \angle ARC = 180^\circ - 52^\circ = 128^\circ$ ($\angle$ sum of Δ)

Therefore, we have $\angle ECR = 128^\circ \div 2 = 64^\circ$.

$\angle CDE = \angle ECR = 64^\circ$ ($\angle$ in alt. segment)



40. D

Method 1

Let p and q be the x-coordinates of P and Q respectively.

Putting $y = -3x + 8$ into $9x^2 + 9y^2 - 18x + 2ky - 72 = 0$, we have

$$9x^2 + 9(-3x + 8)^2 - 18x + 2k(-3x + 8) - 72 = 0$$
$$9x^2 + 9(9x^2 - 48x + 64) - 18x + 2k(-3x + 8) - 72 = 0$$
$$90x^2 + (-6k - 450)x + (16k + 504) = 0 \text{ .... } (*)$$

So, p and q are the roots of (*).

By the relations between roots and coefficients, we have $p + q = -\dfrac{-6k - 450}{90} = \dfrac{k + 75}{15}$.

The x-coordinate of the mid-point of $PQ = \dfrac{p + q}{2} = \dfrac{k + 75}{30}$

Note that the x-intercept of the straight line $3x + y - 8 = 0$ is $\dfrac{8}{3}$.

$$\frac{k + 75}{30} = \frac{8}{3}$$
$$k = 5$$

Method 2

Let O and M be the centre of the circle and the mid-point of PQ respectively.

$$9x^2 + 9y^2 - 18x + 2ky - 72 = 0$$
$$x^2 + y^2 - 2x + \frac{2k}{9}y - 8 = 0$$

The coordinates of $O = \left(-\dfrac{-2}{2}, -\dfrac{1}{2}\left(\dfrac{2k}{9}\right)\right) = \left(1, -\dfrac{k}{9}\right)$

Note that the x-intercept of the straight line $3x + y - 8 = 0$ is $\frac{8}{3}$.

So, the coordinates of M are $\left(\frac{8}{3}, 0\right)$.

Note that $OM \perp PQ$. (line joining centre and mid-pt. of chord $\perp$ chord)

The slope of $OM \times$ The slope of $PQ = -1$

$$\frac{0-\left(-\frac{k}{9}\right)}{\frac{8}{3}-1} \times \left(-\frac{3}{1}\right) = -1$$

$$k = 5$$

41. C

Since I is the in-centre of ΔOAB, the x-axis bisects $\angle AOB$.

So, the slope of OA and the slope of OB are opposite numbers of each other.

Let A' and B' be A and B such that the slope of OA' is positive and the slope of OB' is negative.

Let m be the slope of OA'.

Let a and b be the x-coordinates of A' and B' respectively.

Then, the y-coordinates of A' and B' are ma and $-mb$ respectively.

Note that A' lies on L.

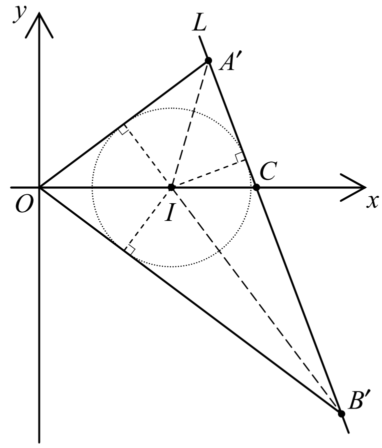
$$117a + 44(ma) - 6720 = 0$$

$$a = \frac{6720}{117 + 44m}$$

Note that B' lies on L.

$$117b + 44(-mb) - 6720 = 0$$

$$b = \frac{6720}{117 - 44m}$$



Let C be the point of intersection of L and the x-axis.

Then, the coordinates of C are $\left(\frac{2240}{39}, 0\right)$.

$$\text{The area of } \Delta OA'C + \text{The area of } \Delta OB'C = \text{The area of } \Delta OA'B'$$

$$\frac{1}{2}\left(\frac{2240}{39} - 0\right)\left(\frac{6720m}{117+44m} - 0\right) + \frac{1}{2}\left(\frac{2240}{39} - 0\right)\left(0 - \frac{-6720m}{117-44m}\right) = 2688$$

$$\frac{1120}{39}\left(\frac{6720m}{117+44m} + \frac{6720m}{117-44m}\right) = 2688$$

$$2800m[(117-44m) + (117+44m)] = 39(117+44m)(117-44m)$$

$$655200m = 39(13689 - 1936m^2)$$

$$1936m^2 + 16800m - 13689 = 0$$

$$(4m-3)(484m+4563) = 0$$

$$m = \frac{3}{4} \quad \text{or} \quad -\frac{4563}{484} \quad \text{(rejected)}$$

Therefore, the coordinates of A' and B' are $\left(\frac{224}{5}, \frac{168}{5}\right)$ and $(80, -60)$ respectively.

$$OA' = \sqrt{\left(\frac{224}{5} - 0\right)^2 + \left(\frac{168}{5} - 0\right)^2} = 56$$

$$OB' = \sqrt{(80-0)^2 + [(-60)-0]^2} = 100$$

$$A'B' = \sqrt{\left(80 - \frac{224}{5}\right)^2 + \left[(-60) - \frac{168}{5}\right]^2} = 100$$

Since $\Delta OA'I$, $\Delta OB'I$ and $\Delta A'B'I$ have equal heights, we have

The area of $\Delta OA'I$: The area of $\Delta OB'I$: The area of $\Delta A'B'I = OA' : OB' : A'B'$

Note that the sum of the areas of $\Delta OA'I$, $\Delta OB'I$ and $\Delta A'B'I$ is equal to the area of $\Delta OA'B'$.

Therefore, we have

$$\text{The area of } \Delta A'B'I = \text{The area of } \Delta OA'B' \times \frac{A'B'}{OA' + OB' + A'B'} = 2688 \times \frac{100}{56+100+100} = 1050$$

Thus, the area of ΔABI is 1050.

42. D

The number of groups consisting of exactly 5 students that can be formed is the number of ways to choose 5 from the 24 students.

The number of groups consisting of exactly 0 girls that can be formed is the number of ways to choose 5 from the 11 boys.

The number of groups consisting of exactly 1 girl that can be formed is the number of ways to choose 1 from the 13 girls and choose 4 from the 11 boys.

$$\begin{aligned}\text{The required number} &= C_5^{24} - C_5^{11} - C_1^{13}C_4^{11} \\ &= 37\ 752\end{aligned}$$

43. A

$$\begin{aligned}\text{The required probability} &= \frac{8}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{2}{7} \\ &= \frac{2}{15}\end{aligned}$$

44. B

Let σ marks be the standard deviation of the examination scores; and
s be the standard score of the girl.

$$\begin{cases} 60 + 2.5\sigma = 70 \text{......(1)} \\ 60 + s\sigma = 65 \text{......(2)} \end{cases}$$

$(1) \times s - (2) \times 2.5$: $\quad 60s - 150 = 70s - 162.5$

$s = 1.25$

45. B

I ✗ Subtracting 2 from each number of the set subtracts 2 from the median.

Dividing each resulting number by 2 makes the median become $\frac{1}{2}$ times the original value.

The median of the new set of numbers $= \frac{m-2}{2} = \frac{m}{2} - 1$

II ✓ Subtracting 2 from each number of the set does not make any change to the range.

Dividing each resulting number by 2 makes the range become $\frac{1}{2}$ times the original value.

The range of the new set of numbers $= \frac{r}{2}$

III ✗ Subtracting 2 from each number of the set does not make any change to the variance.

Dividing each resulting number by 2 makes the variance become $\frac{1}{2^2}$ times the original value.

The variance of the new set of numbers $= \frac{v}{2^2} = \frac{v}{4}$

Candidates' Performance

Paper 1

Candidates generally performed better in Section A than in Section B.

Section A (1)

Question Number	Performance in General
1	Very good. About 90% of the candidates were able to simplify the given formula and answer in positive index. Common Mistakes: - Some candidates confused the index law. For example： $(a^p)^q = a^{p+q}$ 、 $(a^p)^q = a^{p^q}$ or $a^p a^q = a^{p-q}$
2	Very Good. About 75% of the candidates were able to use w as the subject correctly. Common Mistakes: - Few candidates made mistakes in the calculation of $u(2 - w)$. For example: $u(2 - w) = 2u - 2w$
3 (a) (b)	Good. Over 60% of the candidates were able to factorize the given equations. Common Mistakes: - Some candidates were unable to factorize the expression to its simplest. For example: $(3x - 5y)(6x + 10y)$ Few candidates divided the coefficients by two as the factorized coefficient. For example: $18x^2 - 50y^2 = \left(\frac{18}{2}x - \frac{50}{2}y\right)\left(\frac{18}{2}x + \frac{50}{2}y\right) = (9x - 25y)(9x + 25y)$ Fair. Many candidates were unable to write down the expression after factorization. Common Mistakes: - Some candidates incorrectly combined the remainder while taking out the common factor. For example: $3(3x - 5y) - 2(3x - 5y)(3x + 5y) = (3x - 5y)(3x + 5y + 3 - 2)$ Suboptimal method: - Few candidates were unable to use the result of (a) to solve the question and this method was not recommended as it was inefficient.

4 (a)	Very good. About 80% of the candidates were able to write down the approximate value required. Common Mistakes: - Few candidates were unable to notice the condition ('rounding up') in the question. For example: 16.7346 = 16.7
(b)	Very good. Over 70% of the candidates were able to write down the approximate value required by the question. Common Mistakes: - Few candidates ignored the condition ('three decimal places') in the question. For example: 16.7346 = 16.7
(c)	Very good. Over 85% of the candidates were able to write down the approximate value required by the question. Common Mistakes: - Few candidates mistakenly thought that "rounding to the nearest integer" meant rounding from the "units digit" to the "tens digit". For example: 16.7346 = 20
5	Fair. More than half of the candidates were unable to obtain the total price. Common Mistakes: - Most candidates ignored that the number of pencil sharpeners and that of hole punchers were equal when the question was describing the price. Therefore, candidates were unable to calculate the total price required after writing out the system of equations. For example: $\begin{cases} x+5y+5z=363\text{K} \ (1) \\ 3x+2y+2z=322\text{K} \ (2) \end{cases}$ - Few candidates assumed that the price of a pencil sharpener and a hole puncher were equal. For example: $\begin{cases} x+10y=363\text{K} \ (1) \\ 3x+4y=322\text{K} \ (2) \end{cases}$ - Few candidates carelessly assumed that the question was asking for the total price of a pencil case, a pencil sharpener and a hole puncher, so they calculated the wrong answer. For example: 68 + 59 = 127

6 (a)	Very good. About 90% of the candidates were able to find the range of x. Common Mistakes: - Few candidates were unable to properly handle the coefficients of the other terms when rounding the coefficient of x to the nearest integer. For example: $\frac{5x+22}{3} \geq 2(x+3) \quad \Rightarrow \quad 5x+22 \geq 6x+6$ - Few candidates neglected to change the inequality sign when changing the coefficient of x to a positive number. For example: $-x \geq -4 \quad \Rightarrow \quad x \geq 4$
(b)	Good. About 60% of the candidates were able to find the number of positive integers that satisfied the compound inequality. Common Mistakes: - Some candidates were unable to examine the question carefully and did not answer with the number of positive integers which satisfied the compound inequalities. Example: The required numbers are 3 and 4. Suboptimal method: Few candidates copied the answer to (a) completely or did not mention the range of the inequality at all because one of the inequalities was the same as the one in (a). The former was too time-consuming and the latter resulted in incomplete answers. A better way to write the answer is to simply state that it follows the previous question or by (a), and then write the conclusion $2.25 \leq x \leq 4$, so the required number is 2.

7	Fair. Many candidates were unable to obtain $\angle CDE$. Common Mistakes: - Few candidates incorrectly assumed $\angle EAC = 90°$ because AE looked like perpendicular to AC in the diagram. - Few candidates were unable to infer that BD is the diameter and $\angle BED = 90°$ based on the fact that BD was the perpendicular bisector of AC. Suboptimal method: Some candidates used angles as descriptions only. This method does not explicitly indicate which angles the equation is calculating. For example: $180° - 32° - 32° - 90° = 26°$ Few candidates illustrated certain angles by proving congruent triangles, which is a time-consuming method.

8 (a)	Very good. Few candidates were unable to find the median, the range, or the inter-quartile range from the statistical graph. Common Mistakes: - Some candidates neglected to mention that the units were in thousands of dollars. For example: Median: 53 - Few candidates thought that the range and the inter-quartile range were intervals. For example: range: 32-86, inter-quartile range: 41-65
(b)	Fair. More than half of the candidates were unable to explain why the monthly rent of at least 25% of the stores in the mall had increased. Common Mistakes: - Some candidates incorrectly assumed that the question was examining candidates on percentage change based on key words such as '%' and 'increase'. For example: $\frac{13}{53} \times 100\% = 24.53\%$, 24.53 < 25, so disagreed.
9 (a)	Fair. Many candidates were unable to find the ratio of the height of the cone to the radius of the sphere correctly. Common Mistakes: - Some candidates overlooked that there are 9 spheres. - Few candidates took the question to be "1 : 4" when it was the ratio of the "height of a cone" to the "radius of a sphere". - Few candidates incorrectly assumed that the ratio of the volume of a cone to that of a sphere was the ratio of the cube of their radii. For example: $V_s : V_c = (r_s)^3 : (r_c)^3$, where V_s and V_c are representing the volumes of the sphere and the cone respectively, while r_s and r_c are representing the radii of the sphere and the cone respectively.
(b)	Fair. Over 60% of the candidates were unable to find the total surface area of the cone. Common Mistakes: - Some candidates misquoted the radius of the sphere when substituting the value of the radius. For example: $\pi r^2 + \pi rl = 12 \times 12 \times \pi + 12 \times 60 \times \pi$ - Few candidates incorrectly assumed that the surface area of the sphere given in the question was the total surface area of the 9 spheres. For example: $9 \times 4\pi r^2 = 576\pi$ - Few candidates were unable to notice that the cone in this question was a solid cone and therefore neglected to calculate the area of the bottom of the cone. For example: Total surface area: $\pi rl = 36 \times 60 \times \pi$

Section A (2)

Question Number	Performance in General
10 (a)	Very good. Most candidates were able to find f(x). Common Mistakes: - Few candidates incorrectly substituted the value of the unknown they assumed when writing f(x). For example: Let $f(x) = k_1 + k_2(x+2)^2 \Rightarrow k_1 = -61$ and $k_2 = 4$， $\therefore f(x) = 4 - 61(x+2)^2$
(b)	Good. Many candidates were able to find the solution of $f(x) = 13x$. Common Mistakes: - Few candidates mistook $(x+2)^2 = x^2 + 2^2$ and therefore failed to find the solution of $f(x) = 13x$.
11 (a)	Good. About 60% of the candidates were able to find the values of a and b. Common Mistakes: - Some candidates were unable to assume the values of the three data in the way $20 + a$ and $40 + b$. For example: $\frac{25+a+27+29+34+36+38(3)+42+44+2b+47+51+55}{16} = 39$ Suboptimal method: - Few candidates directly identified $a = 6$ and used this to find $b = 7$. There are three possible values for$a$. It is not a good idea to rule out the other two possibilities without specifying any reasons.
(b)	Poor. Most candidates were unable to score full marks in this question. Common Mistakes: Most candidates simply wrote their answers without providing any reasons or used the conditions in the question as reasons. For example: Since there is only one mode, so c = 38 or 47. "There is only one mode" is the condition given in the question, other reasons should be included in order to reduce the range of values of c, i.e. 38 or 47. The condition given in the question is important, but the reasons hidden behind it are what candidates should have answered.
(c)	Poor. Over 75% of the candidates were unable to write all possible standard deviations. Common Mistakes: - Some candidates wrote only one standard deviation and claimed it as the maximum standard deviation without providing any justification. In this question there are two possible values of c and therefore two possible standard deviations. If only one standard deviation is calculated, then reasons must be givento explain why it is the maximum possible standard deviation before it can be claimed as the maximum possible standard deviation. However, there are only two possibilities in this question, so it is less time consuming to calculate the two possible values directly and to compare.

12 (a)	Fair. Many candidates were unable to write down the quotient when f(x) is divided by $x^2 - x - 2$. Common Mistakes: - Some candidates were unable to understand the relationship between f(x) and the divisor, the quotient and the remainder. i.e. $f(x) = g(x) \times q(x) + r(x)$, where g($x$), q($x$) and r($x$) are the divisor, the quotient and the remainder respectively.
(b)	Poor. Most candidates were unable to state clearly how many rational roots f(x) has. Common Mistakes: - Some candidates were unable to factorize f(x). - Some candidates were able to use "factorization", "discriminant" or "sum / product of roots formula" but failed to explain clearly the meaning of the results obtained by these three methods. For example: $(3x + 7)(2x^2 - 2x + 1) = 0 \Rightarrow x = \frac{-7}{3}$ and $\Delta < 0$ (or $x = \frac{1 \pm \sqrt{-1}}{2}$) $\therefore$ f(x) has only one rational root. Candidates did not specify further whether the resulting root was a rational root, or even did not write down what $\Delta < 0$ represents.
13 (a) (i)	Good. Many candidates were able to demonstrate $\Delta ACF \sim \Delta BDF$. Suboptimal method: - When using angles to prove similar triangles, candidates only need to prove that the angles of any two of the three pairs of angles are equal, and the third pair of angles can be ignored. Few candidates even used the angles of two pairs of angles to deduce that the angles of the third pair of angles are also equal, which is time-consuming and unnecessary.
(a) (ii)	Poor. Most candidates were unable to prove $\Delta ABC \sim \Delta EDC$. Suboptimal method: - Some candidates proved $\Delta ABC \sim \Delta EDC$ by using the fact that quadrilateral $ABCD$ is a cyclic quadrilateral, but the question required the use of (a)(i), meaning that the result of (a)(i) had to be used, not any of the statements that appeared in the answer to (a)(i). This question requires the use of the pair of similar triangles proved in (a)(i) in order to satisfy the requirements of the question.
(b) (i)	Poor. About 80% of the candidates were unable to explain correctly that ΔCDE is a right-angled triangle. Common Mistakes: - After finding the length of CE, few candidates assumed that the length of DE was 15 and then used Pythagoras Theorem to explain that ΔCDE is a right-angled triangle. However, the question did not provide the length of DE. For example: $CE = 20$，$20^2 + 15^2 = 25^2 = CD^2$, according to Pythagoras Theorem ΔCDE is a right-angled triangle.

(b) (ii)	Poor. About 80% of the candidates were unable to calculate the area of ΔABC correctly. Common Mistakes: - Some candidates applied $\frac{1}{2}(AC)(BC)\sin\angle ACB$ to find the area, however they used approximate values while calculating $\sin\angle ACB$. ΔABC is a right-angled triangle and it would be more appropriate to calculate the exact value. For example: $\cos\angle ACB = \frac{52}{65} \Rightarrow \angle ACB = 36.9° \Rightarrow \frac{1}{2}(65)(52)\sin 36.9° = 1014$ Suboptimal method: - Few candidates applied Heron's formula to calculate the area, but in this question ΔABC is a right-angled triangle, so dividing the base times the height by 2 to calculate the area can save calculation time.
14 (a)	Fair. Many candidates were unable to write the equation of the circle. Common Mistakes: - Some candidates set the equation of circle as $x^2 + y^2 + Dx + Ey + F = 0$, but were unable to notice that the coordinates of the centre was lying on the straight line $2x + 5y - 161 = 0$ .Therefore, they were unable to find the equation of the circle. - Some candidates thought that the centre of the circle lie on the line AB and assumed that AB was the diameter.
(b) (i)	Fair. More than half of the candidates were unable to describe the geometric relationship between Γ and the line segment AB. Common Mistakes: - Few candidates were using the word "each other" when describing the geometry of two lines. However, in this question, Γ is a straight line that extends in both directions. For example: Γ and the line AB are perpendicular to each other. - Few candidates simply gave the answer 'perpendicular bisector', a sentence without a subject and a clause which did not clearly describe the geometric relationship between Γ and AB.
(b) (ii) (1)	Poor. About 80% of the candidates were unable to find the correct distance between points D and G. Common Mistakes: - Some candidates were unable to find the coordinates of point G correctly in (a), resulting in a wrong distance. Few candidates described the geometric relationship between and the line AB incorrectly in (b) and therefore failed to solve the equation of straight line correctly. This is a type of question that must follow from the results of the previous question. Any errors in the process or results of the previous question will result in failure to score full marks in this question, so the preceding

	discussion should be checked carefully for errors. - Candidates confused the concept of " Γ intersects with x-axis" and " Γ intersects with y-axis". For example: Let the coordinates of D be $(x, 0)$.
(b) (ii) (2)	Poor. Most candidates were unable to find the ratio of the areas of the two triangles correctly. <u>Common Mistakes:</u> - Candidates want to use the ratio relationship to solve the question, but the two triangles they want to compare in this question are not similar triangles and the ratio of their areas is not the ratio of the square of the side lengths. For example: Area of ΔABE : Area of $\Delta ADE = AB^2 : AD^2$. - Graphing is not necessary, but sometimes graphing can enhance understanding of the question and help finding ideas. Some candidates failed to understand the position of the two triangles on the rectangular coordinate system and made some incorrect judgments. For example: $AE \perp DE$ or $AE \perp BE$.

Section B

Question Number	Performance in General
15 (a)	Poor. About 50% of the candidates were able to find the answer to this question. <u>Common Mistakes:</u> - Few candidates confused the symbols of 'permutation' and 'combination'. For example: C_5^6 write as P_5^6
(b)	Poor. Over 80% of candidates were unable to obtain an answer to this question. <u>Common Mistakes:</u> - Many candidates mistook (a) and (b) to be complementary events and incorrectly assumed that 1 − (a) is what (b) was requiring. - Some candidates were unable to enumerate all the situations that satisfied the question. For example: $\dfrac{C_5^6 \times C_1^4 + C_4^6 \times C_2^4 + C_3^6 \times C_3^4 + C_2^6 \times C_4^4}{C_6^{11}}$
16 (a)	Good. About 50% of the candidates were able to obtain A(1). <u>Common Mistakes:</u> - Some candidates confused with the formula for arithmetic sequence. For example: $a + (6 - 1)d = 246$
(b)	Poor. Most candidates were unable to answer this question correctly. <u>Common Mistakes:</u> - Some candidates incorrectly assumed that G(1)G(2)G(3) ··· G(k) is a sum of geometric sequence. - Some candidates ignored the fact that log0.9 is a negative number and did not change the sign of the inequality when doing the calculation. For example: $\log 0.9[\mathrm{A}(1) + \mathrm{A}(2) + \mathrm{A}(3) + \cdots + \mathrm{A}(k)] > \log 2023$ $\mathrm{A}(1) + \mathrm{A}(2) + \mathrm{A}(3) + \mathrm{L} \ + \mathrm{A}(k) > \dfrac{\log 2023}{\log 0.9}$
17 (a)	Fair. About 40% of the candidates were able to find the range of values of m. <u>Common Mistakes:</u> - Some candidates were able to associate the range of values of m with the discriminant when they saw the question, but few candidates incorrectly included the case that the discriminant was zero. For example: $(-10m + 12)^2 - 4(m^2 + 1)(25) \geq 0$

(b)	Poor. Most candidates were unable to obtain the coordinates of P. Common Mistakes: - Some candidates were unable to understand the meaning of the question, leading them to assume a value for m and then find the coordinates of P. For example: Let $m = \frac{1}{6}$
18 (a)	Fair. More than 55% of the candidates were unable to express c in terms of k. Common Mistakes: - Candidates incorrectly squared the negative sign of x^2 when substituting $2k$. For example: $(-2k)^2 + 6k(2k) + c = 0$ - Some candidates confused the formula of 'sum of roots' with 'product of roots'. For example: Let the x-coordinate of B be p, $2k + p = \frac{6k}{-1} = -6k$, $\therefore$ $p = -8k$，$c = 2k \times (-8k) = -16k^2$.
(b)	Fair. About 70% of the candidates were unable to find the coordinates of Q by the method of completing square. Common Mistakes: - Some candidates confused the positive and negative signs when performing the method of completing square. For example: $-x^2 + 6kx + 8k^2 = -(x^2 + 6kx + 9k^2) + 8k^2 + 9k^2$ - Some candidates overlooked the need to square the constant term when performing the method of completing square. For example: $-x^2 + 6kx + 8k^2 = -(x^2 + 6kx + 3k) + 8k^2 - 3k$ - Few candidates not only used k, but also c, when expressing the coordinates of Q For example: $Q(3k, 9k^2 + c)$ - Few candidates applied the "method of completing square" and confused the positive and negative signs of the vertices. For example: $-(x - 3k)^2 + k^2$, so the coordinates of Q are $(-3k, k^2)$ - Few candidates failed to notice that the question required the use of the method of completing square and used other methods to find the coordinates of Q.
(c) (i)	Poor. About 80% of the candidates were unable to obtain the coordinates of R. Suboptimal method: - Some candidates obtained $g(x)$ before finding the coordinates of R. This question did not require candidates to write $g(x)$, so this method was more time-consuming.

(c) (ii)	Poor. Most candidates were unable to explain that A, B, Q and R are not concyclic correctly. Common Mistakes: - Some candidates were unable to notice that the orthocentre of ΔABQ was on the straight line $x = 3k$ and thus were unable to calculate $k = 6$. - The product of the slopes of two perpendicular lines is -1. Few candidates applied this concept incorrectly. For example: $m_{AQ} = k$ ， $m_{BQ} = -k$ ， $\frac{k}{-1} = -k$ ， $\Rightarrow AQ \perp BQ$

19 (a)	Poor. Most of the candidates could not write down a complete proof Common Mistakes: - Some candidates confused the names of the line segments or angles in their answers. For example: $\frac{IK}{\sin \angle ILK} = \frac{IL}{\sin \angle IKL} \Rightarrow \sin \angle IKL = \frac{KL \sin \angle LIK}{IK}$ - Few candidates assumed on their own that $IJ \perp KL$, however, according to the question, L is any point on IJ. - The sine theorem refers to the proportional relationship between the angle and the opposite side in "a triangle". Some candidates confused this concept. For example: $\frac{IL}{\sin \angle IKL} = \frac{JL}{\sin \angle JKL}$
(b) (i)	Poor. Most candidates failed to find the length of VC. Common Mistakes: - Many candidates incorrectly assumed that the point of projection of V onto plane $ABCD$ was the intersection of AC and BD. - Some candidates incorrectly assumed that the $VC \perp CD$.
(b) (ii) (1)	Poor. Most candidates were unable to find the ratio of the length of VQ to the length of CQ. Few candidates were able to perceive the need to use the results of question (a) but were unable to find the appropriate triangle.
(b) (ii) (2)	Poor. Most of the candidates were unable to find the volume of the pyramid. Most of them failed to notice that AQ is the perpendicular bisector of PR and therefore encountered difficulties in applying the formula for the volume of the pyramid.
(b) (ii) (3)	Poor. Most candidates were unable to determine the relationship between the magnitude of the two angles in the question. For the question of finding the angle of two planes, candidates can try to write down the angles they are looking for in the question. In this question, for example, the required angles are $\angle VAQ$ and $\angle CAQ$. Even if the angles are not found, there is still a chance of getting marks.

Candidates' Performance

Paper 2

The paper consisted of 45 multiple-choice items. The mean score was 22.7. Post-examination analysis revealed the following:

1. Candidates' performance was good on Items 1, 3, 4, 5, 11, 14 and 44. Over 70% of the candidates answered them correctly.
2. Candidates' performance on Items 15, 16, 21, 32, 33, 34, and 40 was unsatisfactory. Less than 30% of the candidates gave the correct answers.

Common mistakes made by candidates:

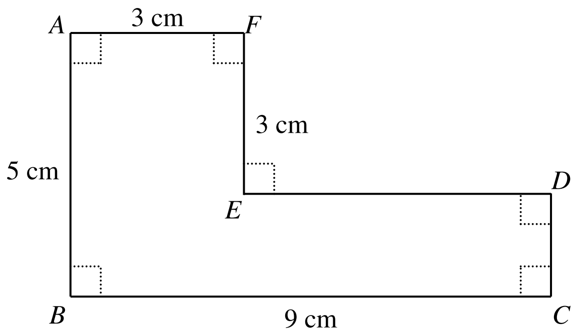
1. In Item 12, many candidates confused the relationship between area and length in similar graphs and hence wrongly gave Option A as the answer.

   Q.12 The scale of a map is 1 : 25 000. If the actual area of a lake is 2.4 km^2, then the area of the lake on the map is

A.	9.6 cm^2 .	(38%)
B.	23.04 cm^2 .	(7%)
*C.	38.4 cm^2 .	(48%)
D.	92.16 cm^2 .	(7%)

2. In Item 15, many candidates mistakenly thought that taking the maximum of the range of all the measured lengths on the diagram would result in the largest area and hence wrongly gave Option D as the answer.

Q.15 In the diagram, *ABCDEF* is a hexagon in which all the measurements are corrected to the nearest cm. Let x cm^2 be the actual area of the hexagon. Find the range of values of x.



A.	$15.5 < x < 38.5$	(27%)
*B.	$17.25 < x < 37.25$	(17%)
C.	$20.75 < x < 34.75$	(25%)
D.	$23.25 \leq x < 31.25$	(31%)

3. In Item 16, many candidates thought that the lateral height of the pyramid was 15 cm and hence wrongly gave Option C as the answer.

Q.16 The base of a solid pyramid is a rhombus and its side length is 24 cm. The sides of the pyramid are known to be congruent. If the height and volume of the pyramid are 15 cm and 1 920 cm^3 respectively, then the total surface area of the pyramid is

A.	720 cm^2 .	(13%)
B.	816 cm^2 .	(21%)
C.	1 104 cm^2 .	(38%)
*D.	1 200 cm^2 .	(28%)

4. In Item 21, many candidates thought that E was the centre of the circle and hence wrongly gave Option C as the answer.

Q.21 AC is a radius of circle $ABCD$. AC intersects BD at point E. If $\overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{CD} = 3 : 6 : 2$, then $\angle BEC =$

A.	50° .	(12%)
B.	60° .	(16%)
C.	120° .	(43%)
*D.	130° .	(29%)

5. In Item 27, the equation of the graph on a rectangular coordinate system was generally a weaker area for candidates. Many candidates failed to understand the graph of circle C on the rectangular coordinate system in this item and hence gave wrong answers.

Q.27 The equation of circle C is $\frac{x^2}{4}+\frac{y^2}{4}+8x-12y+39=0$. The origin is defined as O. Which of the following(s) is/are correct?

I. O is outside of C.

II. C is in the second quadrant.

III. For any two points P and Q on C, $\angle POQ$ must be less than a right angle.

*A.	I only	(33%)
B.	II only	(23%)
C.	I and III only	(30%)
D.	II and III only	(14%)

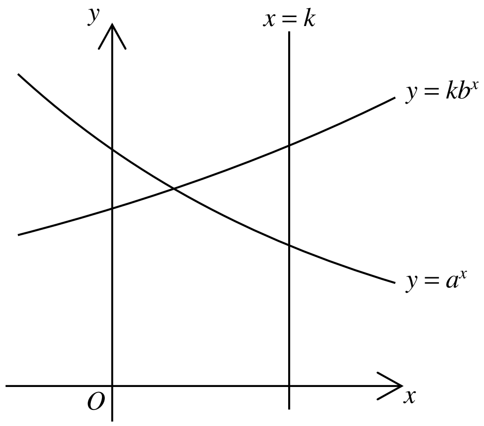
6. In Item 32, many candidates wrongly thought that $y = kb^x$ is an ascending curve and thus $k > 1$ and hence wrongly gave Option C as the answer.

Q.32 The Cartesian coordinate system shows the graph of $y = a^x$, $y = kb^x$ and straight line $x = k$, in which a, b and k are both real numbers. Which of the following(s) is/are correct?

I. $k < 1$

II. $a < b$

III. $a < kb$



A. I and II only (17%)

B. I and III only (21%)

C. II and III only (43%)

*D. I, II and III (19%)

7. In Item 33, many candidates confused the positive and the negative signs while applying the relationship between the roots of quadratic equation and its coefficients, and hence wrongly gave Option C as the answer.

Q.33 Let α and β be the roots of the quadratic equation $x^2 + mx + n = 0$, where m is a constant and n is a non-zero constant. If $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{m}{2} - 2$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} =$

A. $\frac{2m - m^2}{2}$. (20%)

*B. $\frac{6m - m^2}{2}$. (30%)

C. $\frac{m^2 - 2m}{2}$. (28%)

D. $\frac{m^2 - 6m}{2}$. (22%)

8. In Item 34, many candidates wrongly thought that the common ratio of a_n is a negative number and given that $a_6\,a_7\,a_8\,a_9\,a_{10} = -64$, there are no irrational numbers in a_n, Therefore, they mistook I be an incorrect statement and III be a correct one, and hence wrongly gave Option C as the answer.

Q.34 Let a_n be the n th term of a geometric sequence. If $a_6\,a_7\,a_8\,a_9\,a_{10} = -64$ and for any positive integer n, the sum of the first n terms of the series is positive, then which of the following(s) must be true?

I. There are some irrational numbers in the sequence.

II. The second term of the sequence is a negative number.

III. The common ratio of the sequence is less than -1.

*A.	I and II only	(23%)
B.	I and III only	(23%)
C.	II and III only	(36%)
D.	I , II and III	(18%)

9. In Item 40, "The x-coordinate of the midpoint of two points is equal to the sum of the two x-coordinates and then divided by 2." Some candidates have ignored the step of dividing by 2 and hence wrongly gave Option C as the answer.

Q.40 Let k be a constant. Straight line $3x + y - 8 = 0$ intersects circle $9x^2 + 9y^2 - 18x + 2ky - 72 = 0$ at points P and Q. If the midpoint of PQ is lying on the x-axis, find k.

A.	-155	(16%)
B.	-115	(22%)
C.	-35	(32%)
*D.	5	(30%)